SIR ARTHUR LEWIS COMMUNITY COLLEGE
ACADEMIC YEAR (2024/2025) – SEMESTER ONE
END OF SEMESTER EXAMINATION

COURSE TITLE : **CALCULUS I**

COURST CODE : **MAT146**

LECTURER : **ANTONIA GOODMAN**
JOHN ESTEPHANE

DATE : **Wednesday 11th December 2024**

TIME : **1.00 – 3.00**

DURATION : **2 HRS**

STUDENT ID# : **…….**

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. Write your student's ID number on all the work you hand in.
2. This paper consists of **TWO (2)** sections**.**
3. Answer ***all*** the questions in section A and B.
4. Write in BLACK or BLUE pen**.** ***(NO WRITING IN PENCIL)***
5. Show all calculations and working.
6. **Only NON-PROGRAMMABLE calculators are permitted**.
7. Do not use correction fluid.

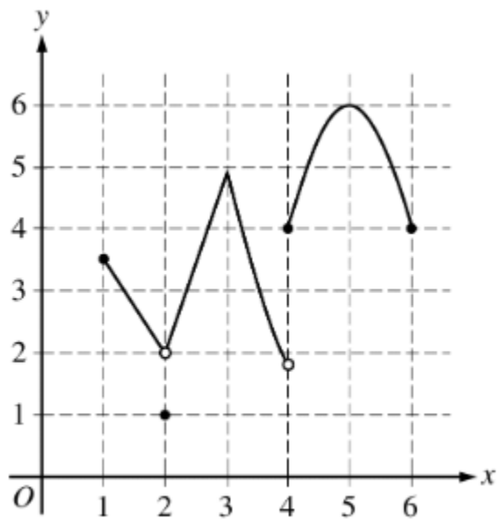
Question	Student's mark	Max Score
1-20		40
1		11
2		16
3		13
4		10
5		5
6		5
TOTAL		**100**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Section A

Circle the letter for the best answer.

The graph of a function f is shown below. Use the graph to answer questions 1 to 5.



Graph of f

1. What is the $\lim_{x \to 3} f(x)$
   (A) 0
   (B) Does not exist
   (C) 4
   (D) 5

2. What is the value of f (2)?
   (A) 2
   (B) 1
   (C) Undefined
   (D) 5

3. What type of discontinuity exist at $x = 2$?
   (A) Jump discontinuity
   (B) Removable discontinuity
   (C) Infinite discontinuity
   (D) Essential discontinuity

4. What type of discontinuity exists at x = 4?
   (A) Jump discontinuity
   (B) Removable discontinuity
   (C) Infinite discontinuity
   (D) Essential discontinuity

5. If $y = \frac{2x+3}{x+2}$, then $\frac{dy}{dx} =$

   a. $\frac{6x-1}{(x+2)^2}$
   b. $\frac{2x^2+5x-3}{(x+2)^2}$
   c. $\frac{1}{(x+2)^2}$
   d. $\frac{7}{(2x+3)^2}$

6. $\lim_{x \to \infty} \frac{16x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} =$
   a. 4
   b. 2
   c. DNE
   d. 0

7. If $y = x \sin 2x$, then $\frac{dy}{dx} =$
   a. $2x \cos 2x$
   b. $x \cos 2x$
   c. $2xcos\ 2x + \sin 2x$
   d. $x(2x \cos 2x + \sin 2x)$

8. A curve has first derivative $2x - 1$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(2, -3)$?
   a. $y = 5x - 3$
   b. $y = x^2 + 1$
   c. $y = x^2 - x - 5$
   d. $y = 2x^2 + 3x - 5$

9. Describe the continuity of $f(x) = \begin{cases} x^2 - 9, & x < 1 \\ 3x - 1, & x \geq 1 \end{cases}$
   a. F(x) is continuous
   b. F(x) has jump discontinuity at $x = 1$
   c. F(x) has removable discontinuity at $x = 1$
   d. F(x) has an infinite discontinuity at x$x = 1$

10. Given that the $\lim_{x \to 0} f(x) = 6$, $\lim_{x \to 0} g(x) = -8$ and $\lim_{x \to 0} h(x) = 10$. Find the $\lim_{x \to 0} \frac{g(x)}{h(x)-f(x)}$
    a. -2
    b. 2
    c. -3
    d. DNE

11. The positive value of **a** for which $\int_0^a x\ dx = 8$ is
    a. 1
    b. 2
    c. 3
    d. 4

12. The positive value of z for which $\int_z^4 2x\ dx = 9$ is
    a. 3
    b. 4
    c. 6
    d. 8

13. What is $\int \sqrt{x^3}$ dx

a. $\frac{3}{2}\sqrt{x^5}$ + C
b. $\frac{2}{5}\sqrt{x^5}$ + C
c. $\frac{2}{5}x^{\frac{1}{5}} + C$
d. $\frac{3}{2}\sqrt{x} + C$

14. The area of the region bounded by the curve $y = x^2$ and the lines x = 1 and x = 2 and the x axis is:
a. $\frac{7}{3}$ square units
b. $\frac{50}{3}$ square units
c. $\frac{52}{3}$ square units
d. 8 square units

15. What is the maximum or minimum point for the curve $y = x^2 - 2x + 4$?
a. A minimum at $(1, 3)$
b. A maximum at $(1, 3)$
c. A minimum at $(1, -3)$
d. A maximum at $(1, -3)$

16. Given that $\frac{dy}{dx} = 5\sin x - 8\cos x$, the indefinite integral is given by
a. $5\cos x - 8\sin x + C$
b. $-5\cos x + 8\sin x + C$
c. $5\cos x + 8\sin x + C$
d. $-5\cos x - 8\sin x + C$

17. What is $\int 5x^3\, dx$?
a. $\frac{5}{4}x^4 + C$
b. $15x^2 + C$
c. $\frac{5}{2}x^2 + C$
d. $6x + C$

18. $\int (4x^3 + \sec^2 x)\, dx =$
a. $x^4 + \tan x + C$
b. $x^4 + \sec^2 x$
c. $12x^2 - \tan^2 x + C$
d. $12x^2 - \sin x + C$

19. What is $\int (\frac{1}{x} - sin\, x)\, dx$
a. $-\frac{1}{x^2} + \cos x + C$
b. $-\ln|x| - cos\, x + C$
c. $\ln|x| + \cos x + C$
d. $\ln|x| - \sin x + C$

20. What is the value of the definite integral? $\int_2^4 x^3\, dx$
a. 15
b. 60
c. 256
d. 64

SECTION B

Write your responses in the space provided. Show all the necessary work.

1. Answer the following questions for the piecewise function $f(x)$ described below.

$$f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2 & x = 1 \\ 6x - 7, & x > 1 \end{cases}$$

(a) $f(1) =$

(b) $\lim_{x \to 1^-} f(x) =$

(c) $\lim_{x \to 1^+} f(x) =$

(d) $\lim_{x \to 1} f(x) =$

[6 marks]

(b). Find $\lim_{x \to 2} \frac{x^2+3x-5}{x-2}$.

[5 marks]

2. The equation of the curve, C is given by $y = 2x^3 - 9x^2 + 12x$

   a) Show that the curve, C, has two stationary points [10 marks]

b) Determine the nature of each stationary point. [6 marks]

3. a) Find $\int (10 + 2x)^5 \mathrm{dx}$ [4 marks]

(b) Find the value of $\int_0^2 (3x^2 + 3x + 1) dx$ [4 marks]

(c) The function $f(x)$ is such that $f'(x) = 4x^3 + 6x^2 + 2x - k$, where k is a constant. Given that $f(0) = 5$ and $f(1) = 12$, find the function $f(x)$.
[5 marks]

4. A particle moves along a straight line with an acceleration of $a = 10 - 8t \quad t \geq 0$. At time t = 0 the particle is at the point O and its velocity is 6 ms^{-1}.

a) Find an expression for the velocity of the particle at time t s. [3 marks]

b) Find an expression for the displacement of the particle from O at time t s
[3 marks]

c) Determine the time the particle comes to rest instantaneously and the distance of the particle from O at this time. [5 marks]

5. Calculate the area under the curve of a function, f(x) = 7 – x^2, the x-axis and the ordinates $x = -1$ and $x = 2$. [5 marks]

6. Find the volume of the solid formed when the area bounded by the curve $y = 3x + x^2$, the x-axis, and the lines x = 2 and x = 3 is rotated through 360^0 about the x -axis. Find the volume generated. [5marks]

END OF EXAMINATION